BARYON NUMBER TRANSPORT IN A COSMIC QCD-PHASE TRANSITION

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ABSTRACT

We investigate the transport of baryon number across phase boundaries in a putative first order cosmic QCD-phase transition. Two independent phenomenological models are employed to estimate the baryon penetrability at the phase boundary: chromoelectric flux tube models; and an analogy to baryon-baryon coalescence in nuclear physics. Our analysis indicates that baryon transport across phase boundaries may be order of magnitude more efficient than other work has suggested. We discuss the substantial uncertainties involved in estimating baryon penetrability at phase boundaries.

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1. Introduction

The possibility of a cosmic first order QCD-phase transition [1,2] has generated considerable interest. Such a phase transition would occur when the universe cools to a temperature of order $T \sim 100\text{MeV}$. During a *first order* QCD phase transition we would expect macroscopic separation of phases, so that macroscopic bubbles of "quark-gluon plasma" phase coexist in rough pressure equilibrium with a "hadron" phase of distinct color-singlet particles.

It has been speculated that a first order QCD-phase transition in the early universe could produce observable remnants. In particular, possible formation of baryon number inhomogeneities during such a first order transition has been considered extensively [1,2,3,4,5]. Baryon number inhomogeneities which persist to the epoch $T \sim 100\text{keV}$ could affect primordial nucleosynthesis yields. Primordial nuclear abundances in such inhomogeneous models can be very different from those in a homogeneous standard Big Bang. There has been extensive research on the effects of baryon inhomogeneities on the epoch of primordial nucleosynthesis [3,4,5,6]. Observationally inferred primordial abundances may allow us to constrain aspects of inhomogeneity production in cosmic phase transitions.

Inhomogeneities could also lead to the formation of stable or metastable strange-quark nuggets [1,7]. It has been suggested that stable nuggets could be a natural candidate for dark matter. The existence of nuggets could also have implications for the epoch of primordial nucleosynthesis [8]. It has even been suggested that primordial black holes [9] and primordial magnetic fields [10] could originate in a first order QCD-phase transition.

Remnants from a cosmic QCD-phase transition, such as baryon inhomogeneities, strange quark matter nuggets, primordial black holes, and primordial magnetic fields, could only be produced if the transition is *first order*. Recent QCD lattice calculations seem to favor a higher order transition [11]. However, the numerical calculations are still restricted to rather small lattices and thus do not correctly model the continuum limit. Another complication in lattice models comes from uncertainty in the numerical treatment of light quarks [12]. A definitive answer on the order of the QCD-transition probably will come only with the development of a future generation of computers.

If the QCD-transition is of first order, there will be an associated macroscopic separation of phases. Crudely, we can describe each component as either a "deconfined" quark-gluon plasma phase, or a "confined" hadron phase. During the phase transition these components will be able to coexist in thermodynamic and chemical equilibrium at a coexistence temperature, T_c. Bonometto and Pantano [13] have given a good review of many of the phase transition issues.

In the early universe an idealized scenario for a first order QCD-phase transition might be as follows [5]. Initially the universe is at high temperature and in the quark-gluon plasma phase. The net baryon number reside entirely in the quark-gluon plasma and is distributed homogeneously. Eventually, the cosmic expansion will cool the universe to temperature T_c and small bubbles of a hadronic phase appear for the first time. Subsequently, the cosmic expansion will require a continuous conversion of quark-gluon plasma to hadron phase. The rate of conversion between quark-gluon plasma and hadron phase is determined by the requirement that the universe be kept at (or very near) the coexistence temperature, T_c. The phase transition is completed when all the quark-gluon plasma has been converted to hadron phase. At this point, all the baryon number resides in the hadron phase and is distributed homogeneously. This scenario assumes a universe which is in complete thermal and chemical equilibrium during the transition.

In reality, a cosmic first order QCD-phase transition will necessarily result in deviations from thermal and chemical equilibrium. The magnitude of these deviations will depend on the efficiency of heat and baryon transport during the phase transition.

There are two mechanism for heat transport: hydrodynamic flow and neutrino heat conduction [2]. In the case of hydrodynamic flow, the quark-gluon plasma phase will maintain a slightly higher pressure than the hadronic phase. This pressure gradient will cause a fluid flow, and an associated energy flow, from the quark-gluon plasma to the hadron phase. Quarks and antiquarks in the quark-gluon plasma will convert into color singlet mesons on the phase boundary. This conversion of quarks and antiquarks into mesons will involve details of strong interaction physics.

In the case of neutrino heat conduction, the quark-gluon plasma will maintain a slightly higher temperature than the hadronic phase. The temperature gradient will cause

a heat flow from the quark-gluon plasma to the hadron phase. Heat will be transported most efficiently by neutrinos. This is because neutrinos have relatively long mean free paths. Heat transport by neutrinos does not involve strong interaction physics at the phase boundary, in contrast to the case of heat transport by hydrodynamic flow. Which heat transport mechanism is the dominant one depends on the equation of state of the phases, the physical properties of the phase boundary, and the geometry of the phase boundary [14]. Heat transport by neutrinos could be favored over heat transport by hydrodynamic flow if there is an inhibition in the formation of mesons at the phase boundary.

The transport of baryon number from the quark-gluon plasma into the hadron phase necessarily involves strong interaction processes at the phase boundary. This is because weakly interacting neutrinos can not carry baryon number.

In the present paper we estimate the baryon number penetrability at the phase boundary. This baryon number penetrability we denote as Σ_h. Specifically, Σ_h is the probability that a baryon which approaches the phase boundary from the hadronic phase, such as a proton, neutron, or baryonic resonance, will dissociate into quarks and pass over into the quark-gluon plasma. Alternatively, we could estimate the probability that a quark which approaches the phase boundary from the quark-gluon plasma will form a color singlet baryon at the phase boundary. We denote this probability as Σ_q. The thermal averages of the probabilities, $\langle\Sigma_h\rangle$ and $\langle\Sigma_q\rangle$, are related by detailed balance [5]

$$\kappa f_{q-\bar{q}}\langle\Sigma_q\rangle = f_{b-\bar{b}}\langle\Sigma_h\rangle \ . \tag{1}$$

Here $f_{q-\bar{q}}$ is the excess quark flux, i.e. the flux of quarks minus the flux of antiquarks. Analogously, $f_{b-\bar{b}}$ is the excess baryon flux.

The dimensionless quantity κ will take values between $\kappa = 1/3$ and $\kappa = 1$. In the limit where $\kappa = 1/3$, baryon number predominantly passes over the phase boundary as pre-formed three-quark clusters in the quark-gluon phase. We would expect this limit to obtain for quark-gluon plasmas which are highly correlated. The value $\kappa = 1/3$ then accounts for the fractional baryon number (1/3) of a single quark. In the limit where $\kappa = 1$, baryons will be predominantly formed by single energetic quarks which cross over

the phase boundary into the hadron phase. We will estimate the efficiency of this process in detail in Section 2.

The maximum possible value of $\langle \Sigma_h \rangle$ is unity, i.e. all baryons which approach the phase boundary from the hadron phase dissociate into three free quarks and pass into the quark-gluon plasma. A value of $\langle \Sigma_h \rangle = 1$ is possible when there is no energy threshold associated with this process. However, the quarks in the baryon have to "rearrange" themselves into quasi-free quarks at the phase boundary. This rearrangment process might involve an energy barrier. Thus it is conceivable that $\langle \Sigma_h \rangle$ is well below unity.

It should be mentioned that baryon number inhomogeneities can form during a first order QCD-phase transition even for maximum baryon number penetrability, $\langle \Sigma_h \rangle = 1$. When bubbles of quark-gluon plasma are small in size (i.e., towards the end of the phase transition) phase boundaries will move at high velocities. If phase boundaries move rapidly compared to baryon number transport rates, then the chemical equilibrium in baryon number across the phase boundaries may break down. In essence, a shrinking bubble of quark-gluon plasma may shrink so fast that the baryon number inside of it is "trapped" and turned into hadrons in situ.

In this way baryons could be concentrated in the shrinking quark-gluon plasma bubbles. Note that this process starts from chemical equilibrium. Even in equilibrium, baryon number preferentially resides in the quark-gluon phase [4]. Net baryon number densities in the quark-gluon plasma can exceed net baryon number densities in the hadron phase by up to several orders of magnitude, even if both phases are in chemical equilibrium with each other. Thus, towards the end of the phase transition a large fraction of the net baryon number may still reside in shrinking bubbles of quark-gluon plasma.

Between the middle and the end of the phase transition the typical phase boundary velocity will increase continuously with time. The velocity for which baryon number transport is too slow to establish chemical equilibrium near the phase boundary is determined by bulk properties of the phases and the baryon number penetrability, $\langle \Sigma_h \rangle$. A low value for $\langle \Sigma_h \rangle$ implies an early drop-out of chemical equilibrium and thus might lead to the formation of very high-amplitude baryon number inhomogeneities. Such a scenario might lead to the formation of strange quark matter nuggets or other remnants.

In Section 2 we will estimate $\langle \Sigma_h \rangle$ with the help of phenomenological chromoelectric flux tube models. Such a calculation has been done previously by Sumiyoshi *et al.* [15]. We will find results quite different than those obtained by Sumiyoshi *et al.*. In Section 3 we employ an analogy to baryon-baryon coalescence in nuclear physics in order to give an independent estimate of $\langle \Sigma_h \rangle$. We give conclusions in Section 4.

2. Chromoelectric Flux Tube Models

Chromoelectric flux tube models provide us with a phenomenological understanding for the formation of hadrons from initially "free" quarks. These models assume the existence of a chromoelectric field between any two oppositely colored quarks. The chromoelectric field strength is assumed to be constant in magnitude and independent of the separation of the quarks. These fields can be thought of as being confined to a tube of constant width. This is referred to as flux tube. Results of lattice QCD justify the assumption of such a flux tube [16]. Chromoelectric flux tube models are successful in explaining observations of processes such as $e^+e^- \mapsto hadrons$ [17,18]. These models can also provide qualitative, and quantitative, information on the spectrum and masses of mesonic and baryonic resonances [17,19]. Flux tube models have been employed to estimate meson evaporation rates from quark-gluon plasmas which are thought to form in heavy-ion collisions [20] and to estimate baryon number penetrabilities at the phase boundary between a quark-gluon plasma and a hadron phase in a putative cosmic first order QCD-phase transition [15].

In the Lund-model [18] the process of $e^+e^- \mapsto hadrons$ is seen as an annihilation of the e^+e^--pair into a quark-antiquark-pair ($q\bar{q}$). A chromoelectric flux tube is formed between the $q\bar{q}$-pair. The maximum separation between the quark and the antiquark or, equivalently, the maximum flux tube length (x_m) depends on the initial energy of the e^+e^--pair (E). Both quantities are related by

$$E = \sigma x_m \ , \tag{2}$$

where $\sigma \approx 0.177\ \mathrm{GeV}^2$ is the string tension of the flux tube. Hadrons are then formed by successive $q\bar{q}$-pair creations within the chromoelectric field of the flux tube.

This process has its analogue in QED. It is known that there is a finite probability for pair creation of an e^+e^--pair in a strong external electric field [21]. The electron and positron will arrange themselves in such a way that the electric field is partially screened and weakened. The rest mass of this "real" e^+e^--pair can be provided by the decrease in the external electric field energy.

In QCD a chromoelectric field between a $q\bar{q}$-pair of colors $(1\bar{1})$ can be screened in two different ways: **(a)** by the pair creation of a $q\bar{q}$-pair with the same color as the original $q\bar{q}$-pair; or **(b)** by two successive pair creations of $q\bar{q}$-pairs with colors $(2\bar{2})$ and $(3\bar{3})$. In general, processes **(a)** and **(b)** will happen multiple times within a long flux tube. Pair creation will stop when the energy in the chromoelectric field is exhausted. At this point, all quarks and antiquarks are arranged in such a way that only residual chromoelectric fields remain. This arrangement corresponds to a configuration of several color-singlet hadrons. Thus the energy of the chromoelectric field has been converted to rest mass and kinetic energy of hadrons in the final state. Note that process **(a)** corresponds to the formation of two mesons and process **(b)** corresponds to the formation of a baryon-antibaryon pair.

Consider now the phase boundary between a quark-gluon plasma and a hadron phase in a cosmic first order QCD-phase transition. We can identify three microscopic mechanisms for the transport of baryon number from the quark-gluon plasma over the phase boundary into the hadron phase. **(I)** An energetic quark of color (1) passes over the phase boundary into the hadron phase. The quark's color is screened by the nearest plasma antiquark of color $(\bar{1})$. A flux tube forms between this quark-antiquark pair and the flux tube "decays" via two successive pair creations of quarks with colors $(2\bar{2})$ and $(3\bar{3})$. A color-singlet baryon (consistent of three quarks with colors 123) forms in the hadron phase and three antiquarks (colors $\bar{1}\bar{2}\bar{3}$) pass back into the quark-gluon phase. **(II)** An energetic quark of color (1) passes over the phase boundary into the hadron phase and its color is screened by two close by plasma quarks of colors (2) and (3). The leading quark of color (1) forces (drags) the two screening quarks into the hadron phase. A baryon is formed in the hadron phase. **(III)** A pre-formed "baryonic cluster" of three quarks which initially resides in the quark-gluon plasma passes over the phase boundary as a unit into the hadron phase. The energy of this cluster exceeds the threshold energy for the formation

of a baryon in the hadron phase, and the result is again the formation of a baryon. To some extent mechanisms **(II)** and **(III)** can be thought of describing the same process. The efficiency of these processes is hard to estimate since we are uncertain about the pre-formation probability of "baryonic clusters" within the quark-gluon plasma.

In the following we estimate the efficiency of mechanism **(I)** with the help of a chromoelectric flux tube model. Since we neglect baryon number transport by mechanisms **(II)** and **(III)** we can only obtain a lower limit for the baryon number penetrability, $\langle\Sigma_q\rangle$. We can relate this lower limit on $\langle\Sigma_q\rangle$ to a lower limit on $\langle\Sigma_h\rangle$ if we use detailed balance in equation (1) and take $\kappa = 1$.

We can imagine a quark of color (1) which passes over an idealized discontinuous phase boundary into the hadron phase at time $t = 0$. As already described above, a flux tube will form between this leading quark and a plasma antiquark of color $(\bar{1})$. The flux tube will continuously increase its length and slow the motion of the leading quark until all the kinetic energy of the quark is converted into energy of the flux tube. At this time, t_0, the leading quark will reverse its motion and move back in the direction of the phase boundary.

There is, however, a probability that the flux tube decays by $q\bar{q}$-pair creation before time t_0. We denote the probability that the flux tube decays before time t by $P_d(t)$. This probability satisfies

$$dP_d(t) = k_b V(t)\big(1 - P_d(t)\big)dt + k_m V(t)\big(1 - P_d(t)\big)dt \ , \tag{3}$$

where $V(t)$ denotes the volume of the flux tube at time t and k_m denotes the probability per unit time and unit volume for the decay of the flux tube by a single $q\bar{q}$-pair creation. Similarly, k_b denotes the probability for the decay of the flux tube by two successive $q\bar{q}$-pair creations. In our calculation we will only consider the decay of flux tubes by either a single pair creation of a $q\bar{q}$-pair with colors $(1\bar{1})$ or two successive pair creations of $q\bar{q}$-pairs with colors $(2\bar{2})$ and $(3\bar{3})$. We will not include additional pair creations. This approximation is reasonable for the environment under consideration, since only a very small fraction of thermal quarks within the quark-gluon plasma will have enough energy to create flux tubes

in which multiple $q\bar{q}$-pair creations are possible. The probability, $P_b(t)$, that the flux tube decays via two successive $q\bar{q}$-pair creations before time t satisfies

$$dP_b(t) = k_b V(t)\big(1 - P_d(t)\big)dt \ . \tag{4}$$

Equations (3) and (4) can be solved to yield:

$$P_b(t) = \frac{k_b}{k_b + k_m}\Big(1 - \exp\big(-(k_b + k_m)\int_0^t V(t')dt'\big)\Big) \ . \tag{5}$$

We expect the formation of a baryon in the hadron phase when three conditions are met: **(a)** the flux tube decays via two successive $q\bar{q}$-pair creations; **(b)** the energy of the leading quark exceeds the energy threshold for baryon formation; and **(c)** the flux tube decays while the leading quark still has momentum which is directed away from the phase boundary, i.e. before time t_0. The three antiquarks of colors $(\bar{1}\bar{2}\bar{3})$ will pass back into the quark-gluon plasma since there is not enough energy to produce the rest masses of both a baryon and an antibaryon in the hadron phase.

With the help of this simple prescription we can obtain the baryon formation probability or, equivalently, the baryon number penetrability, if we solve for the dynamics of the quarks during this process. This probability should be given by $P_b(t_0)$ from equation (5) whenever the energy of the leading quark exceeds the threshold for baryon formation. We can approximate the length of the flux tube to be given at any time by the shortest distance between the leading quark and the phase boundary. That is, we assume that the flux tube is perpendicular to the phase boundary.

We denote the perpendicular distance of the leading quark from the phase boundary as $x_\perp$. Additionally, we denote the lateral position coordinate *along* a phase boundary as $x_\parallel$.

Energy conservation (E) and momentum conservation for the momentum component parallel to the phase boundary ($p_\parallel$) yield

$$E = const = \frac{\sigma x_\perp}{(1 - \dot{x}_\parallel^2)^{1/2}} + \frac{m_q}{(1 - \dot{x}^2)^{1/2}} \ , \tag{6a}$$

$$p_{\parallel} = const = \frac{(\sigma x_{\perp})\dot{x}_{\parallel}}{(1-\dot{x}_{\parallel}^2)^{1/2}} + \frac{m_q \dot{x}_{\parallel}}{(1-\dot{x}^2)^{1/2}} \ . \tag{6b}$$

In these expressions $x^2 = x_{\parallel}^2 + x_{\perp}^2$, while a dot over any quantity indicates a time derivative, and m_q is the quark current mass. These equations imply that the quark velocity parallel to the phase boundary ($\dot{x}_{\parallel}$) is conserved. We can solve the equation of motion and compute the integral $\int_0^{t_0} V(t)dt = \pi\Lambda^2 \int_0^{t_0} x_{\perp}(t)dt$, where Λ is the radius of the flux tube. In the limit of small quark masses $(m_q/E) \mapsto 0$ we obtain

$$\int_0^{t_0} V(t)dt = \frac{\pi\Lambda^2}{2\sigma^2} E^2 \cos\theta \ , \tag{7}$$

with θ the angle of incidence of the leading quark relative to the phase boundary. Using equations (5) and (7) we can determine the baryon formation probability, $P_b(E,\theta) \equiv P_b(t_0)$, for a quark of energy E which approaches the phase boundary at an incident angle θ.

In order to form a baryon in the hadron phase the energy of the leading quark has to exceed a threshold energy. The leading quark has to have enough energy to produce the baryon rest mass of ($m_b \sim 1\ GeV$) and the kinetic energy for the motion of the baryon parallel to the phase boundary. The second contribution to this energy threshold follows from the conservation of velocity parallel to the phase boundary. The energy threshold condition is given by

$$E > E_{th} = \frac{m_b}{\cos\theta} \ . \tag{8a}$$

It is conceivable that the energy threshold exceeds the one given in equation (8a). This is because the initial state includes a quark and an antiquark in the quark-gluon plasma (colors $1\bar{1}$), whereas the final state includes a baryon in the hadron phase and three antiquarks (colors $\bar{1}\bar{2}\bar{3}$) in the quark-gluon plasma. There is an associated average interaction energy for each quark and antiquark which resides in the quark-gluon plasma. We can estimate this average energy if we approximate the quark-gluon plasma with the help of the bag models. Such an approximation gives $E_{int} \approx 3.7T$ [5] for the average energy. The final state in the process of baryon formation includes an additional quark.

It is not known to what extent a cooling of the quark-gluon plasma could provide the interaction energy for the additional quark. We could modify the threshold condition in equation (8a) to be

$$E > E_{th} = \frac{m_b}{cos\theta} + B n_q^{-1} \ . \tag{8b}$$

The correct energy threshold should be between the values given in equations (8a) and (8b).

We can now compute the thermal average of the baryon number penetrability at the phase boundary in a cosmic QCD-phase transition if we assume that mechanism **(I)** is the dominant mechanism for baryon number transport across the phase boundary. We find

$$\langle \Sigma_h \rangle = \frac{1}{f_{b-\bar{b}}} \int_0^{\pi} d\theta \int_{E_{th}}^{\infty} dE \, \frac{dn_{q-\bar{q}}}{dE d\theta} \, \dot{x}_{\perp}^{q}(\theta) \, P_b(E,\theta) \ , \tag{9}$$

where $n_{q-\bar{q}}$ is the excess density in quarks, i.e. the density of quarks minus the density of antiquarks, and $f_{b-\bar{b}}$ is the excess flux in baryons as in equation (1). The differential excess quark number density for quarks in a given energy interval (dE) and in a given interval of incident angles ($d\theta$) is

$$\frac{dn_{q-\bar{q}}}{dE d\theta} = \frac{\mu_q}{T} \frac{g_q}{2\pi^2} \frac{E^2 \exp{(E/T)}}{(\exp(E/T)+1)^2} \sin\theta \ . \tag{10}$$

In this expression the quantities μ_q, g_q, and T are the quark chemical potential, the statistical weight of quarks, and the temperature, respectively.

Equation (10) assumes that an isotropic and massless quark Fermi gas obtains in the quark-gluon plasma during the transition. The statistical weight of quarks, g_q, is the product of the number of colors (3) times the number of relativistic quark flavors (probably 2, the up and down quark) times the number of possible quark spins (2). The excess flux in baryons within the hadronic phase, $f_{b-\bar{b}}$, is given by

$$f_{b-\bar{b}} = \frac{\mu_b}{T} \frac{g_b}{2\pi^2} \left(m_b + T\right) T^2 \exp(-m_b/T) \ . \tag{11}$$

In equation (11) m_b, μ_b, and g_b are the baryon mass, baryon chemical potential, and baryon statistical weight, respectively. Note that $g_b = 4$ and $\mu_b = 3\mu_q$. The component of the quark velocity perpendicular to the phase boundary is simply

$$\dot{x}^q_\perp(\theta) = \cos\theta \ . \tag{12}$$

In order to obtain a quantitative result for $\langle\Sigma_h\rangle$ from equations (5-9) we need to estimate the values for the quantities $a \equiv k_b/(k_m + k_b)$ and $b \equiv \pi\Lambda^2(k_b + k_m)/2\sigma^2$. In the limit, $k_b << k_m$, these quantities can be approximated by $a \approx k_b/k_m$ and $b \approx \pi\Lambda^2 k_m/2\sigma^2$.

In principle the value for the quantity b can be inferred by an extension of the e^+e^--pair creation probability in an electric field in QED. This pair creation probability is accurately known [21]. In the case of QCD the $q\bar{q}$-pair creation probability within a flux tube is

$$b \approx \frac{\pi\Lambda^2 k_m}{2\sigma^2} = f_1\Lambda^2 \sum_{n=1}^{\infty} \frac{1}{n^2} exp\left(-f_2 \frac{m_q^2 n}{\sigma}\right) \ , \tag{13}$$

with f_1 and f_2 numerical factors and m_q the quark mass. The pair creation probability given by equation (13) has been compared to results of $e^+e^- \mapsto hadrons$ experiments. There is disagreement in the literature about the numerical values of f_1 and f_2. In addition, the values for the flux tube width, Λ, and the quark mass, m_q, are not accurately known. In the present analysis we will treat b as a parameter. We assume a value for b between $b = 0.32$ GeV^{-2} [17] and $b = 0.05$ GeV^{-2} [20]. In comparison Sumiyoshi *et al.* [15] used the rather small value of $b = 0.044$ GeV^{-2}.

We can infer the value for the quantity $a \approx k_b/k_m$ (i.e. for the ratio of the probability of flux tube decay via two successive $q\bar{q}$-pair creations to the probability of flux tube decay via a single $q\bar{q}$-pair creation) from experiments of $e^+e^- \mapsto hadrons$. We associate the production of a baryon-antibaryon pair with the decay of a flux tube via two successive $q\bar{q}$-pair creations. Similarly, we associate the production of two mesons with the decay of a flux tube via a single $q\bar{q}$-pair creation. A naive estimate of the quantity a would then be the ratio of the number of baryons to the number of mesons produced in e^+e^--annihilations. This ratio would be approximately $a \approx 1/20$, a value consistent with the small ratio which has been used by Sumiyoshi *et al.*. We will argue that the appropriate

value for a is much larger than $a \approx 1/20$. In fact, we think that a better estimate for this quantity is $a \approx 1/5 - 1/3$. This ratio is also in agreement with a suppression factor of $a \approx 1/5$ for the production of baryons relative to the production of mesons which has been deduced by Casher *et al.* [17].

There are two effects for an enhancement of meson production compared to that for baryon production in accelerator e^+e^--annihilations. First, flux tubes of small length and energy do not provide enough energy for the production of a baryon-antibaryon pair. These flux tubes necessarily have to decay into mesons. Second, baryonic resonances eventually decay into a baryon of lower rest mass. In most cases this decay is accompanied by the production of up to three mesons. Thus the total number of mesons and baryons produced in e^+e^--annihilations does not directly reveal the ratio for the flux tube decay probabilities $a \approx k_b/k_m$ which are relevant to our problem.

In e^+e^--annihilations at center-of-mass energy $E_{CM} = 29$ GeV [22] a shower of hadrons with energies varying between $E = E_{CM}/2$ and $E = m_\pi$ is produced. Baryons, of course, are only formed between energies of $E = E_{CM}/2$ and $E = m_p$, with m_p the proton mass. For an approximate determination of the quantity a it is appropriate to compare the number of produced baryons to the number of produced mesons at fixed baryon and meson energy, $E \geq m_p$. In this way low-energy mesons ($E \leq m_p$) which are produced by the decay of short flux tubes or by the decay of baryonic resonances are not counted. If this is done, a ratio of $a \approx 1/3$ - $1/5$ is found to obtain over a wide range of energies.

There is another effect which might further enhance the relevant value for a in the cosmic QCD-phase transition. The thermal average in equation (9) strongly favors energies slightly above the energy threshold for baryon formation. It is known that baryon production cross sections are normally larger near the threshold energy than at higher energies and, thus, the formation of baryons at the phase boundary might be enhanced compared to that in accelerator e^+e^--annihilations. The existence of a multitude of baryonic resonances in the energy range 1 GeV $< E <$ 2 GeV could also imply an anomalously large effective value for the quantity a in the QCD-phase transition.

We have performed a numerical computation to obtain the thermal average of the baryon number penetrability, $\langle \Sigma_h \rangle$, given by equation (9). Our results are displayed in

Figures 1 and 2. Both figures show the thermal average of the baryon number penetrability times the ratio of probabilities (k_m/k_b). In these figures we plot $\langle\Sigma_h\rangle(k_m/k_b)$ as a function of QCD-phase transition temperature T. The thermal average of the baryon number penetrability, $\langle\Sigma_h\rangle$, is then easily obtained if we multiply the results shown in Figures 1 and 2 by the ratio (k_b/k_m). As discussed above, this ratio is approximately $a \approx (k_b/k_m) \approx 1/5 - 1/3$. In both figures we show results for different values of the parameter $b \approx \pi\Lambda^2 k_m/2\sigma^2$, ranging between $b = 0.4$ GeV^{-2} and $b = 0.05$ GeV^{-2}. In Figure 1 we have assumed the threshold condition of equation (8a), while in Figure 2 we have assumed the threshold condition of equation (8b).

Our results imply that the baryon number penetrability $\langle\Sigma_h\rangle$ may be smaller than unity. The baryon number penetrability is found to range between $\langle\Sigma_h\rangle \sim 0.01$ and $\langle\Sigma_h\rangle \sim 0.1$, depending on the quantities a and b, the threshold condition, and the phase transition temperature. We do not find, however, baryon number penetrabilities as small as $\langle\Sigma_h\rangle \sim 10^{-3}$. Such small values for $\langle\Sigma_h\rangle$ have been suggested by Sumiyoshi *et al.* [15] and Fuller *et al.* [5]. Since our results neglect several mechanisms to form baryons at the phase boundary (i.e. mechanisms **(II)** and **(III)** from above), and since we find mechanism **(I)** to be quite efficient we conclude that values for $\langle\Sigma_h\rangle$ as small as $\langle\Sigma_h\rangle \sim 10^{-3}$ are rather unlikely.

However, we think it is essential to point out inherent uncertainties involved in any flux tube calculation of the baryon number penetrability at the phase boundary between a quark-gluon plasma and a hadron phase. We could argue that such calculations are necessarily oversimplified and that the results are model dependent. Several critical uncertainties and assumptions in these calculations are as follows: the assumption of an idealized discontinuous phase boundary; the assumption of a noninteracting nuclear density environment in the phases; the oversimplified classical treatment of the dynamics of leading and screening quarks; the neglect of other mechanisms to transport baryon number across the phase boundary; and the uncertainties in parameters which describe the flux tube and its decay.

Any serious calculation of the baryon number penetrability at the phase boundary should take into account the finite extension and physical nature of the phase boundary

itself. Unfortunately, not much is known about the phase boundary. The width of the phase boundary should be roughly of the order of a Debye color screening length, $L_D \sim 1$fermi. In comparison the extension of flux tubes formed by energetic quarks ($E \sim 1$ GeV) is of the same order, $L_f \sim 1$fermi. Thus hadronization will most likely occur *within* the phase boundary.

The environment under consideration is at nuclear densities in both phases. This environment is compared to results obtained in accelerators ($e^+e^- \mapsto hadrons$) which effectively operate in "vacuum". It is conceivable that hadronization at the phase boundary is a "collective" process which involves a large number of particles.

A serious calculation of $\langle \Sigma_h \rangle$ should also include well known quantum mechanical principles. In our calculation of the dynamics of the quark position we implicitly assumed a simultaneous knowledge of quark position and momentum. Thus, we treated the motion of the quarks completely classically. This, of course, is not correct for microscopic systems such as two quarks separated by roughly, $L_f \sim 1$fermi. A better estimate of baryon number penetrability would be a solution to the Dirac equation for the motion of a quark near a plane parallel, discontinuous potential distribution. This potential would be located at the phase boundary and, in a more sophisticated treatment, could be taken as linearly increasing in the direction towards the hadron phase. We would have to allow for the decay of the potential, which would simulate the decay of the flux tube. Such a calculation would take better account of the quantum mechanical nature of the problem.

The third and fourth critical points in our list have already been briefly discussed above. We suggested alternative mechanisms for baryon number transport across the phase boundary. We are not able to give estimates for the efficiency of these mechanisms. This is mainly due to a poor knowledge of baryon pre-formation probabilities within the quark-gluon plasma.

Possible ranges for the phenomenological quantities a and b have been given. The exact values which should enter into our calculations are uncertain. We have extracted the value for a and b from $e^+e^- \mapsto hadrons$ experiments, i.e. from environments which are substantially different from the environment in a cosmic QCD-phase transition. There is

some hope that results of heavy-ion collision experiments could remove some of these uncertainties. However, even this environment is significantly different from the environment encountered in the early universe. This is because heavy-ion collisions are performed at large net baryon number densities and under conditions which are far from thermodynamic equilibrium, in contrast to a first order QCD-phase transition in the early universe. In view of these uncertainties we believe that flux tube models can only be used to estimate the order of magnitude of the baryon number penetrability at the phase boundary in a cosmic QCD-phase transition.

3. Baryon-Baryon Coalescence

In this section we will briefly discuss analogies of the problem of baryon number transport across the phase boundary in a cosmic QCD-phase transition to issues in nuclear physics. We will attempt to estimate the baryon number penetrability, $\langle\Sigma_h\rangle$, by employing **(a)** nonrelativistic nucleon-nucleon potentials and **(b)** results of a possible explanation for an unresolved phenomena in deep inelastic scattering experiments.

A phenomenological understanding of the internal structure of baryons can be obtained from bag models (for a review see [23]). In bag models baryons and baryonic resonances are described by a Fermi-"sea" of three quasi-free quarks existing in a perturbed vacuum. This perturbed vacuum is of finite spatial extension and is referred to as the "bag". The size of the bag is of the order of one Fermi, the approximate dimension of a baryon. The energy or rest mass of the baryon is then given by the sum of rest masses and kinetic energies of three quarks, the energy of the perturbed vacuum, and the surface energy associated with the surface between perturbed vacuum (the bag) and "ordinary"vacuum. A macroscopic quark-gluon plasma phase could be approximated by an extension of the MIT-bag model [5]. It would consist of a large number of quarks (N) existing in a macroscopic region of perturbed vacuum, i.e. in a large bag.

Consider the probability $\langle\Sigma_h\rangle$ for a baryon which approaches the phase boundary (from the hadronic phase) to dissociate into its constituents (three quarks) and to pass across the phase boundary into the quark-gluon plasma. This process can be approximated

as the coalescence of a three-quark bag (the baryon) with an N-quark bag (the quark-gluon plasma).

This view suggests the following question. What is the probability for two baryons to form a "di-baryon", i.e for two three-quark bags to coalesce and form a six-quark bag. The answer to this question could be intimately related to the probability, $\langle \Sigma_h \rangle$.

Before we try to exploit the suggested analogy we want to establish an essential difference between the coalescence of two baryons and the coalescence of a baryon with a macroscopic quark-gluon phase. This difference is given by the geometries of the two processes. There are two effects pertaining to the simple understanding of baryons from the MIT-bag model. First, the decrease in bag surface area in the process of baryon-baryon coalescence is smaller than the decrease in surface area in the process of coalescence of a baryon with a quark-gluon phase. The associated decreases in surface energy are thus different for the two processes and we expect a higher probability for the coalescence of a baryon with a quark-gluon plasma than for the coalescence of two baryons. Second, in the formation of a six-quark bag the Pauli-exclusion principle plays an important role. Whenever the two baryons in the initial state include two quarks with the same flavor, color, and spin (the probability for this to happen is roughly 1/2) then one of these quarks would have to occupy an excited state within the final state six-quark bag. An excited state in the six-quark bag, in general, will lie several hundred MeV higher in energy than the ground state. For energetic reasons, this state might not then be accessible to the quark, and the formation of a six-quark bag would be Pauli blocked. In the case of a macroscopic quark-gluon plasma in the early universe we expect a quasi-continuum in energy states, so that Pauli blocking is never complete. For these reasons we expect the probability for the coalescence of two baryons to be smaller than the probability for the coalescence of a baryon with a quark-gluon plasma.

3a. Nuclear Core Potentials

It would be interesting if there was an inhibition in the formation of di-baryons (or six-quark bags) observed in nuclear physics experiments. Such an inhibition could imply an anomalously low value for $\langle \Sigma_h \rangle$. It is common nuclear physics knowledge that the

saturation of nuclear bulk matter can only be explained by the existence of a strong repulsion between nucleons at short separation distances (cf. [24]). To this end a “hard core”, which is an infinitely high potential barrier at a nucleon-nucleon separation distance of $r_c \approx 0.5$F is introduced into two-body nucleon-nucleon potentials. There is also an indication for the repulsion of two nucleons at short separation distances from nucleon-nucleon scattering data and deuteron properties [25].

Unfortunately, there are a great number of phenomenological, nonrelativistic nucleon-nucleon potentials which can explain nucleon-nucleon scattering data and deuteron properties. This multitude of parameter-fitted nucleon-nucleon potentials reflects our inability to deduce the correct nucleon-nucleon potential from first principles. Most of these phenomenological potentials employ a nuclear “hard core”. However, in a few cases potentials have been modified to incorporate a *finite* height potential barrier at short distances which is referred to as a nuclear “soft core”. By how much could the potential barrier between two nucleons at short separation distances be reduced before a conflict with nucleon-nucleon scattering data is inevitable?

Bressel, Kerman, & Rouben [26] modified the Hamada-Johnston nucleon-nucleon potential by replacing the nuclear “hard core”with a “soft core”. They extended the core region to $r_c \approx 0.7$F and introduced a finite potential barrier of height $V_0 \approx 600$ MeV at r_c. Lacombe *et al.* [27] changed the Paris nucleon-nucleon potential by incorporating a finite potential barrier of height $V_0 \approx 200$MeV which extends to a radius of $r_c \approx 0.8$F. In these potentials the “soft core” potential barrier at r_c is, however, spin and isospin dependent. Both potentials are in good agreement with nucleon-nucleon scattering data and deuteron properties.

There are two relevant quantities in nuclear physics which have been well determined experimentally: the root-mean-square radius of the deuteron and the nucleon-nucleon scattering length. Here the nucleon-nucleon scattering length is the square root of the zero energy nucleon-nucleon scattering cross section. It seems that any potential which includes a nuclear “hard core”is unable to explain both quantities simultaneously [28]. Most of these potentials overestimate the root-mean-square radius of the deuteron or, if they explain the root-mean-square radius correctly, they underestimate the scattering

length. An obvious cure should be the introduction of a nuclear "soft core". A nuclear "soft core" would slightly lower the root-mean-square radius of the deuteron. However, it would not affect the nucleon-nucleon cross section at zero energy, i.e. the scattering length. Thus knowledge of the root-mean-square radius of the deuteron and the nucleon-nucleon scattering length might be valuable in obtaining additional information about the nucleon-nucleon potential at short separation distances.

We would like to obtain a numerical estimate for the penetration probability of two baryons, assuming a potential barrier of height, V_0, at a baryon-baryon separation distance of r_c. We will denote this probability by Σ. We can then seek a thermal average of this quantity $\langle\Sigma\rangle$ which is appropriate for a thermal distribution of baryons in the early universe at the QCD-phase transition temperature. As discussed above, this probability should be related to the probability for the coalescence of a baryon with a quark-gluon plasma, $\langle\Sigma_h\rangle$. To compute Σ we have performed a simple quantum mechanical computation to determine the tunneling probability of a nonrelativistic baryon through a barrier of height, V_0, and extension, r_c. In such a tunneling process the six-quark bag would be the intermediate state. Our results obtained in this manner are displayed in Figures 3 and 4. In these figures we show $\langle\Sigma\rangle$ as a function of the QCD-phase transition temperature, T. We have employed different potential barriers V_0 (ranging between 200 MeV and 800 MeV) in our estimates of $\langle\Sigma\rangle$.

Figure 3 assumes a potential barrier width of $d = 2r_c = 1$fermi; whereas Figure 4 assumes $d = 1.4$fermi. It is evident that in order to obtain a baryon-baryon penetration probability as small as $\langle\Sigma\rangle = 0.1$, the potential barrier must be high ($V_0 \gtrsim 600$ MeV) and the temperature must be low. A potential barrier as low as $V_0 \approx 200$ MeV [27] results in a penetration probability of $\langle\Sigma\rangle \sim 0.5$. In view of the increased probability for the coalescence of a baryon with a quark-gluon plasma compared to the coalescence of two baryons, our results could be consistent with $\langle\Sigma_h\rangle = 1$.

3b. The EMC-Effect and Six-Quark Bags

In deep inelastic lepton-nucleus scattering experiments a nontrivial deviation of the cross section for scattering of leptons off big nuclei (e.g. iron) compared to that for scattering of leptons off small nuclei (e.g. deuterium) has been observed. This deviation can not be attributed to differences in the Fermi motions of nucleons (Fermi-gas model) within different nuclei. The effect is known as the EMC-effect [29] and the reader is referred to [30] for a detailed discussion.

It has been suggested that the EMC-effect could be explained by a significant six-quark bag component in nuclei [31,32,33]. Detailed estimates indicate that a six-quark bag component of 16% in ^{3}He-nuclei could explain the scattering data of leptons scattering off ^{3}He [32]. Similarly, a six-quark bag component of 30% in ^{56}Fe-nuclei could account for the data observed in leptons scattering off ^{56}Fe [33]. Note, however, that a potential six-quark bag component in nuclei is not the only explanation for the EMC-effect.

If we assume that a six-quark bag component in ^{3}He is the sole explanation of the EMC-effect, we can obtain a simple estimate for the baryon-baryon penetration probability in nuclei. To this end, we imagine three nucleons confined to the volume of a ^{3}He-nucleus as frequently colliding. The fraction of the time during which we would observe a six-quark bag in ^{3}He, P_6, is then given by the total nucleon-nucleon collision frequency, P_c, times the lifetime of a six-quark bag, τ_6, times the probability for penetration of two nucleons, Σ. We can then write,

$$P_6 \approx P_c \tau_6 \Sigma \ . \tag{14}$$

An estimate of the nucleon-nucleon collision frequency can be obtained by assuming a nucleon of cross sectional area πa^2 moving through a nuclear volume $(4\pi/3)r^3$ at a typical velocity v. The total nucleon-nucleon collision frequency is then simply three times the reciprocal of the time which a single nucleon needs to traverse the whole nuclear volume. The factor of 3 enters the calculation since there exist three distinct ways to form a pair of nucleons in ^{3}He. We can approximate the radius of a ^{3}He-nucleus to be $r \approx 2$fermi, and the radius of a nucleon to be $a \approx 0.5$fermi. An approximate nucleon velocity can be obtained

either from the Heisenberg uncertainty principle, or from the momentum expectation value of a nucleon moving in the mean field of a ^{3}He-nucleus.

If we estimate the nucleon-nucleon collision frequency, P_c, in this manner we can rewrite equation (14) as

$$\Sigma \approx \frac{2}{5} \frac{mr^4}{\hbar a^2} \frac{P_6}{\tau_6} \ , \tag{15}$$

where m is the nucleon mass and $\hbar$ is Planck's constant. For lack of a more sophisticated treatment, we assume that the lifetime of a six-quark bag is $\tau_6 \approx 3 \times 10^{-24}$s, a typical strong interaction time. This is also the time light needs to travel over the dimensions of a nucleon. Finally, if we take $P_6 \approx 0.16$ [32], we can infer a nucleon-nucleon penetration probability Σ. With this set of assumptions we obtain the surprising result that $\Sigma \approx 20$! This value can not be correct, since Σ can not exceed unity. Similarly, we obtain a value for Σ which is above unity if we perform an analogous calculation for the nucleus of ^{56}Fe [33].

Obviously, our assumptions and approximations in this crude estimate must break down. We can imagine two simple ways to resolve the issue. It could be that the existence of a six-quark bag component in nuclei is *not* the sole explanation of the EMC-effect. The actual value for P_6 would then be much smaller than the value given by Pirner & Vary [32] and Carlson & Havens [33]. It is also conceivable that the life time of six-quark bags in nuclei, τ_6, is longer than our estimate $\tau_6 \approx 3 \times 10^{-24}$s. If, however, both of these assumptions are correct, i.e. the six-quark bag component in nuclei is substantial and the life time of such six-quark bags is short, then we have to seek the explanation for the large value of Σ in our crude approximations. In this case our results for Σ would be consistent with a baryon-baryon penetration probability of order unity.

4. Conclusions

We have estimated the efficiency of baryon number transport across the phase boundary between a quark-gluon plasma phase and a hadron phase in a putative cosmic first order QCD-phase transition. Such a transition is likely to form baryon number inhomogeneities. The amplitude of these baryon number inhomogeneities could reach very high values if there is a substantial inhibition for the transport of baryon number across the phase boundary.

In our calculations we have employed phenomenological flux tube models and analogies of the problem of baryon number transport across the phase boundary to issues in nuclear physics. We have discussed appreciable uncertainties in computations of the baryon number penetrability. We have argued that such calculations should only be used to indicate the order of magnitude of baryon number penetrabilities. Our results indicate baryon number penetrabilities at the phase boundary which could be consistent with a maximum possible probability of unity.

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Figure Captions

Figure 1 The thermal average of the baryon number penetrability $\langle \Sigma_h \rangle$ times the probability ratio (k_m/k_b) as a function of temperature T in MeV. This product is shown for four different values of the parameter b, ranging between $b = 0.05$ GeV^{-2} and $b = 0.4$ GeV^{-2}. In this figure we have assumed the energy threshold condition given in equation (8a).

Figure 2 Same as Figure 1 but here we have used the energy threshold condition given in equation (8b).

Figure 3 The thermal average of the baryon-baryon penetration probability $\langle \Sigma \rangle$ as a function of temperature T in MeV. Results are shown for square-wave potential barriers with barrier heights $V_0 = 200$ MeV, 400 MeV, 600 MeV, and 800 MeV. The spatial width of the square-wave barrier is assumed to be $d = 1$fermi.

Figure 4 Same as Figure 3 but here we have assumed a width of the square-wave potential barrier of $d = 1.4$fermi.